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PREDICTION OF TIME-TO-GO FOR A HOMING MISSILE USING BANG-BANG CONTROL¹

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Abstract

The flight time required for a variable-speed homing missile to intercept a zero-acceleration target in two-dimensions is determined by assuming that the missile normal acceleration is bang-bang, that is, maximum normal acceleration followed by zero normal acceleration where the switch time which gives intercept is to be determined. For those cases where intercept does not occur in a reasonable time, the flight time which minimizes the miss distance is used. A tangential acceleration profile is assumed for the missile, that is, constant positive acceleration when thrusting, constant negative acceleration when coasting, and a given engine burn-out time. In this way, the velocity of the missile becomes a known function of time, and the equations of motion can be solved analytically. Then, an algebraic equation for the switch time for intercept or the final time for closest approach can be derived, but it must be solved numerically.

The time-to-go algorithm is tested in a six-degree-of-freedom simulation of a homing missile with a linear-quadratic guidance law where the target performs two maximum normal acceleration maneuvers. At each sample point, the missile velocity vector is projected onto the plane of the line-of-sight vector and the target velocity vector, and the time-to-go is calculated for a planar intercept. This time-to-go is used to calculate the gains for the guidance law. Results show that this time-to-go algorithm improves the performance (miss distance) of the missile for several scenarios relative to the range-over-closing-speed algorithm.

List of Symbols

A	Function defined in Eq. (14)
a_n	Missile normal acceleration (ft/sec ²)
a_t	Missile tangential acceleration (ft/sec ²)
a_1, b_1	Constants defining V_M for thrusting flight
a_2, b_2	Constants defining V_M for coasting flight
B	Function defined in Eq. (16)
d	Miss distance (ft)
k	Thrusting, $k = 1$; coasting, $k = 2$
t	Time (sec)
t_0	Initial time (sec)
t_c	Engine cutoff time (sec)
t_f	Final time (sec)

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t_s	Switch time between $a_n = a_{n_{max}}$ and $a_n = 0$ (sec)
V_M	Missile velocity (ft/sec)
V_T	Target velocity (ft/sec)
X	Inertial X coordinate (ft)
x	Relative X coordinate (ft)
Y	Inertial Y coordinate (ft)
y	Relative Y coordinate (ft)
θ	Missile velocity orientation (deg)
ϕ	Target velocity orientation (deg)

Subscripts

c	Engine cutoff
f	Final
o	Initial
p	Point
s	Switch

Introduction

A guidance law of current interest for bank-to-turn homing missiles is the linear-quadratic guidance law which contains proportional navigation as a particular case (see, for example, Ref. 1). In order to implement this guidance law, an algorithm for predicting time-to-go is needed. The simplest time-to-go formula is range divided by closing speed and is valid for a constant-velocity missile and target on a collision course. This formula has been improved in Ref. 2 by accounting for the missile longitudinal acceleration.

Unfortunately, the linear-quadratic guidance law tends to drive the missile and the target into a homing triangle in which range and closing speed become unobservable. In Ref. 1, a linear-quadratic guidance law for dual control (intercept and estimation enhancement) has been proposed. This guidance rule moves the missile away from the homing triangle improving estimation but making the time-to-go algorithm invalid.

The purpose of this study is to develop a time-to-go algorithm which is valid for intercept geometries which differ greatly from the intercept triangle, such as encountered with dual control. This is accomplished by assuming that the missile normal acceleration history is maximum normal acceleration followed by zero normal acceleration (bang-bang) and computing intercept trajectories. In addition, while the velocity of the target is assumed constant, the velocity of the missile is assumed to vary. Here, the tangential acceleration is assumed a positive constant while thrusting and a negative constant while coasting.

As a first step, the analysis is carried out in two dimensions with the hope that some insight in the three-dimensional problem will be achieved. The resulting time-to-go algorithm is tested in a six-degree-of-freedom simulation by projecting the current missile velocity vector onto the plane of the line of sight and the target velocity. The optimal intercept time is computed in this plane and used as the time-to-go for the linear-quadratic guidance law.

Optimal Intercept Problem

Shown in Fig. 1 is the geometry of the intercept problem. The XY coordinate system represents an inertial frame, and the X axis is along the line of sight at $t = 0$. The constant-velocity target, located at $X_T = X_0$ at $t = 0$, is moving along a straight line which makes an angle ϕ with respect to the X axis. The missile is launched at an angle θ_0 relative to the X axis, and the velocity direction $\theta(t)$ is changed by controlling the normal acceleration $a_n(t)$. If $x = X_T - X_M$ and $y = Y_T - Y_M$, the equations of motion of the engagement in relative coordinates are given by

$$\dot{x} = V_T \cos \phi - V_M \cos \theta \quad (1)$$

$$\dot{y} = V_T \sin \phi - V_M \sin \theta \quad (2)$$

$$\dot{\theta} = a_n / V_M \quad (3)$$

The tangential acceleration history of the missile is assumed to be constant $a_{t_{max}} > 0$ while thrusting, that is, for $t \leq t_c$ where t_c is the known engine cutoff time, and a constant $a_{t_{min}} < 0$ while coasting. As a consequence, the velocity of the missile while thrusting is given by

$$V_M = a_1 + b_1 t, \quad t \leq t_c \quad (4)$$

where

$$a_1 = V_{M_0}, \quad b_1 = a_{t_{max}}. \quad (5)$$

Similarly, the velocity of the missile during the coasting phase is given by

$$V_M = a_2 + b_2 t, \quad t \geq t_c \quad (6)$$

where

$$a_2 = V_{M_0} + (a_{t_{max}} - a_{t_{min}}) t_c, \quad b_2 = a_{t_{min}} \quad (7)$$

The prescribed initial conditions are

$$t_0 = t_{0s}, \quad x_0 = x_{0s}, \quad y_0 = y_{0s}, \quad \theta_0 = \theta_{0s} \quad (8)$$

where the subscript s denotes a specific value. Intercept at the final point requires that

$$x_f = 0, \quad y_f = 0. \quad (9)$$

If θ_0 were free, the control for the minimum-time trajectory would be $a_n = 0$, that is, a straight line. On the other hand, if θ_0 were prescribed, the minimum-time control would be infinite normal acceleration to rotate the velocity vector instantaneously to the above straight-line followed by zero normal acceleration. If a bound were applied to the normal acceleration, the optimal control would become

maximum normal acceleration followed by zero normal acceleration, that is, a bang-bang control. The bang-bang control is used here to generate a minimum-time trajectory, and the flight time is used as a prediction of the time-to-go for linear-quadratic guidance rules.

In the solution of the problem, it is found that a direct intercept can be achieved for $\theta_{ZA} \leq \theta_0 \leq \theta_{MA}$ (see Fig. 2) where θ_{ZA} is the initial angle for the zero-normal-acceleration intercept and θ_{MA} is the initial angle for the maximum-normal-acceleration intercept. For $\theta_0 > \theta_{MA}$, the missile passes in front of the target during the maximum normal acceleration phase. Then, it performs a 360 deg turn before it goes for the intercept. When this happens the minimum time is taken as the time to the point of closest approach. A similar discussion holds for $\theta_0 < \theta_{ZA}$.

In the development of the equations, there are four important times: the initial time t_0 , the final time t_f , the engine cutoff time t_c , and the switch time t_s between $a_n = a_{t_{max}}$ and $a_n = 0$. It is assumed that

$$t_0 \leq t_s \leq t_f \quad (10)$$

so that $a_n = a_{t_{max}}$ all the way if $t_s = t_f$, and $a_n = 0$ all the way if $t_s = t_0$. In the development of the equations, the engine cutoff time is assumed to satisfy the inequality

$$t_0 \leq t_c \leq t_f. \quad (11)$$

Then, if $t_c < t_0$ (coasting all the way), the correct equations can be obtained by setting $t_c = t_0$, and if $t_c > t_f$ (thrusting all the way), t_c is set equal to t_f .

Constant Normal Acceleration

The equations of motion (1) through (3) can be integrated for the case where a_n is constant. These solutions are valid for the cases where $a_n = a_{t_{max}}$ or $a_n = 0$.

Since the missile velocity has the general form $V_M = a_k + b_k t$ where $k = 1$ for thrusting and $k = 2$ for coasting, Eq. (3) can be integrated as

$$\theta = \theta_p + \frac{a_n}{b_k} \ln \left(\frac{a_k + b_k t}{a_k + b_k t_p} \right) \quad (12)$$

The subscript p denotes a generic starting point; it could be the initial point, the engine cutoff point, or the switch point.

Next, with Eq. (12), Eq. (1) can be integrated to yield

$$x = x_p + A(t, a_n, k, t_p, \theta_p) - A(t_p, a_n, k, t_p, \theta_p) \quad (13)$$

where

$$A(t, a_n, k, t_p, \theta_p) = V_T \cos \phi t - \frac{2b_k(a_1 + b_1 t)^2}{4b_k^2 + a_n^2} \cdot \left\{ \cos \left[\theta_p + \frac{a_n}{b_k} \ln \left(\frac{a_1 + b_1 t}{a_1 + b_1 t_p} \right) \right] + \frac{a_n}{2b_k} \sin \left[\theta_p + \frac{a_n}{b_k} \ln \left(\frac{a_1 + b_1 t}{a_1 + b_1 t_p} \right) \right] \right\} \quad (14)$$

Integration by parts is used to obtain the second term in Eq. (14).

Finally, Eq. (2) for y is integrated in the same way as Eq. (1) and leads to

$$y = y_p + B(t, a_n, k, t_p, \theta_p) - B(t_p, a_n, k, t_p, \theta_p) \quad (15)$$

where

$$B(t, a_n, k t_p, \theta_p) = V_T \sin \phi t - \frac{2b_k(a_k+b_k t)^2}{4b_k^2 + a_k^2} \cdot \left\{ \sin \left[\theta_p + \frac{a_n}{b_k} \ln \left(\frac{a_k+b_k t}{a_k+b_k t_p} \right) \right] - \frac{a_n}{2b_k} \cos \left[\theta_p + \frac{a_n}{b_k} \ln \left(\frac{a_k+b_k t}{a_k+b_k t_p} \right) \right] \right\} \quad (16)$$

Zero-Normal-Acceleration Intercept

In this section, the zero-normal-acceleration intercept (straight-line intercept $\theta = \theta_{ZA}$) is derived. It is the dividing line between the $a_n \geq 0$ ($\theta_o \leq \theta_{ZA}$) intercepts and $a_n \leq 0$ ($\theta_o \geq \theta_{ZA}$) intercepts. For the time being, the cutoff time, which is known, is assumed to satisfy the inequality constraint $t_o < t_c < t_f$.

If $a_n = 0$, Eqs. (12), (13) and (15) can be applied at t_c and t_f and the results combined to yield

$$\theta_f = \theta_o \quad (17)$$

$$x_f = x_o + A(t_c, 0, 1, t_o, \theta_o) - A(t_o, 0, 1, t_o, \theta_o) + A(t_f, 0, 2, t_c, \theta_o) - A(t_c, 0, 2, t_c, \theta_o) \quad (18)$$

$$y_f = y_o + B(t_c, 0, 1, t_o, \theta_o) - B(t_o, 0, 1, t_o, \theta_o) + B(t_f, 0, 2, t_c, \theta_o) - B(t_c, 0, 2, t_c, \theta_o) \quad (19)$$

For intercept, $x_f = y_f = 0$ so that Eqs. (17) through (19) involve two unknowns: t_f and $\theta_o \equiv \theta_{ZA}$. These equations can be solved for $\cos \theta_{ZA}$ and $\sin \theta_{ZA}$ as

$$\cos \theta_{ZA} = \frac{V_T \cos \phi t_f + x_o}{[a_1 + \frac{1}{2} b_1 (t_c - t_o)](t_c - t_o) + [a_2 + \frac{1}{2} b_2 (t_f - t_c)](t_f - t_c)} \quad (20)$$

$$\sin \theta_{ZA} = \frac{V_T \sin \phi t_f + y_o}{[a_1 + \frac{1}{2} b_1 (t_c - t_o)](t_c - t_o) + [a_2 + \frac{1}{2} b_2 (t_f - t_c)](t_f - t_c)}$$

which, in turn, can be squared and added to obtain the single equation for t_f

$$\left\{ \frac{V_T \cos \phi t_f + x_o}{[a_1 + \frac{1}{2} b_1 (t_c - t_o)](t_c - t_o) + [a_2 + \frac{1}{2} b_2 (t_f - t_c)](t_f - t_c)} \right\}^2 +$$

$$\left\{ \frac{V_T \sin \phi t_f + y_o}{[a_1 + \frac{1}{2} b_1 (t_c - t_o)](t_c - t_o) + [a_2 + \frac{1}{2} b_2 (t_f - t_c)](t_f - t_c)} \right\}^2 = 1 \quad (21)$$

Once t_f is known, θ_{ZA} follows from Eqs. (20). If $t_c \geq t_f$, the proper equations can be obtained from Eqs. (20) and (21) by setting $t_c = t_f$. On the other hand, for $t_c \leq t_o$, set $t_c = t_o$.

Maximum-Normal-Acceleration Intercept

The maximum-normal-acceleration intercept is the dividing line between intercept trajectories and closest-point-of-approach trajectories. It is possible to determine this trajectory by applying Eqs. (12), (13), and (15). However, a system of two equations in two unknowns results, and while it can be solved efficiently with careful coding, a simpler approach is used to determine whether an intercept is possible.

Once θ_{ZA} is known, the actual θ_o can be compared with θ_{ZA} to determine the sign of a_n : $\theta_o \leq \theta_{ZA}$, $a_n \geq 0$ or

$\theta_o \geq \theta_{ZA}$, $a_n \leq 0$. Then, the equations of motion are integrated by Euler integration for $a_n = a_{nmax}$ over the entire trajectory. At the point where $x_f = 0$, the value of y_f is investigated to determine whether an intercept can occur. If the missile passes behind the target ($y_f > 0$), an intercept can be achieved, and if the missile passes ahead of the target ($y_f < 0$), no intercept is possible. Should the missile turn have a small enough radius that $x_f \neq 0$ after a reasonable time, an intercept is deemed possible.

Intercept Trajectories

For an intercept, the switch time is assumed to satisfy the inequality $t_o < t_c < t_f$ because $t_c = t_o$ is the zero-normal-acceleration intercept, and $t_c = t_f$ is the maximum-normal-acceleration intercept. There are four possible configurations for an intercept trajectory depending on the value of the known engine cutoff time, that is, $t_c \leq t_o$, $t_o \leq t_c \leq t_s$, $t_s \leq t_c \leq t_f$, $t_c \geq t_f$. The equations for $t_c \leq t_o$ can be obtained from those for $t_o \leq t_c \leq t_s$ by setting $t_c = t_o$, and the equations for $t_c \geq t_f$ can be obtained from those for $t_s \leq t_c \leq t_f$ by setting $t_c = t_f$. Hence, only two sets of equations need to be derived.

$t_o \leq t_c \leq t_s$

For this case, Eqs. (12), (13), and (15) can be applied at t_c and t_s and combined to yield

$$\theta_s = \theta_c + \frac{a_n}{b_2} \ln \left(\frac{a_2 + b_2 t_s}{a_2 + b_2 t_c} \right), \quad \theta_c = \theta_o + \frac{a_n}{b_1} \ln \left(\frac{a_1 + b_1 t_c}{a_1 + b_1 t_o} \right) \quad (22)$$

$$x_s = x_o + A(t_c, a_n, 1, t_o, \theta_o) - A(t_o, a_n, 1, t_o, \theta_o) + A(t_s, a_n, 2, t_c, \theta_c) - A(t_c, a_n, 2, t_c, \theta_c) \quad (23)$$

$$y_s = y_o + B(t_c, a_n, 1, t_o, \theta_o) - B(t_o, a_n, 1, t_o, \theta_o) + B(t_s, a_n, 2, t_c, \theta_c) - B(t_c, a_n, 2, t_c, \theta_c) \quad (24)$$

Along the straight-line part ($a_n = 0$), Eqs. (12), (13), and (15) become

$$\theta_f = \theta_o \quad (25)$$

$$x_f = x_o + A(t_f, 0, 2, t_s, \theta_s) - A(t_s, 0, 2, t_s, \theta_s) \quad (26)$$

$$y_f = y_o + B(t_f, 0, 1, t_s, \theta_s) - B(t_s, 0, 2, t_s, \theta_s) \quad (27)$$

Because of Eqs. (14) and (16), the explicit forms of Eqs. (26) and (27) are given by

$$x_f = x_o + \{V_T \cos \phi - [a_2 + \frac{1}{2} b_2 (t_f - t_s)] \cos \theta_s\} (t_f - t_s) \quad (28)$$

$$y_f = y_o + \{V_T \sin \phi - [a_2 + \frac{1}{2} b_2 (t_f - t_s)] \sin \theta_s\} (t_f - t_s) \quad (29)$$

The solution process is to vary t_s until x_f and y_f are both zero. This is accomplished by setting $x_f = 0$ and solving Eq. (28) analytically for t_f . Eq. (27) is a quadratic equation and has two roots. The correct root is the smallest value of t_f which is larger than t_s . Then, the solution for t_f is substituted into Eq. (29) where, because t_s is guessed, $y_f \neq 0$. Hence, t_s is varied until $y_f = 0$. In this way, it is only necessary to solve one equation in one unknown.

$$t_s \leq t_c \leq t_f$$

If the engine cutoff time is between the switch time and the final time, Eqs. (12), (13), and (15) lead to

$$\theta_c = \theta_o, \quad \theta_s = \theta_o + \frac{a_n}{b_1} \ln \left(\frac{a_1 + b_1 t_s}{a_1 + b_1 t_o} \right) \quad (30)$$

$$x_c = x_o + A(t_s, a_n, 1, t_o, \theta_o) - A(t_o, a_n, 1, t_o, \theta_o) \\ + A(t_c, 0, 1, t_s, \theta_s) - A(t_s, 0, 1, t_s, \theta_s) \quad (31)$$

$$y_c = y_o + B(t_s, a_n, 1, t_o, \theta_o) - B(t_o, a_n, 1, t_o, \theta_o) \\ + B(t_c, 0, 1, t_s, \theta_s) - B(t_s, 0, 1, t_s, \theta_s) \quad (32)$$

For the coast, the corresponding equations are

$$\theta_f = \theta_c \quad (33)$$

$$x_f = x_c + A(t_f, 0, 2, t_c, \theta_c) - A(t_c, 0, 2, t_c, \theta_c) \quad (34)$$

$$y_f = y_c + B(t_f, 0, 2, t_c, \theta_c) - B(t_c, 0, 2, t_c, \theta_c) \quad (35)$$

The explicit forms of Eqs. (34) and (35) are similar to those of Eqs. (28) and (29). Hence, the same solution process can be used.

Closest-Point-of-Approach Trajectories

For $a_n = a_{n,\max}$ over the entire trajectory and for $t_s \leq t_c \leq t_f$, Eqs. (12), (13), and (15) can be applied at t_c and t_f , and combined to obtain

$$\theta_f = \theta_c + \frac{a_n}{b_2} \ln \left(\frac{a_2 + b_2 t_f}{a_2 + b_2 t_c} \right), \quad \theta_c = \theta_o + \frac{a_n}{b_1} \ln \left(\frac{a_1 + b_1 t_c}{a_1 + b_1 t_o} \right) \quad (36)$$

$$x_f = x_o + A(t_c, a_n, 1, t_o, \theta_o) - A(t_o, a_n, 1, t_o, \theta_o) \\ + A(t_f, a_n, 2, t_c, \theta_c) - A(t_c, a_n, 2, t_c, \theta_c) \quad (37)$$

$$y_f = y_o + B(t_c, a_n, 1, t_o, \theta_o) - B(t_o, a_n, 1, t_o, \theta_o) \\ + B(t_f, a_n, 2, t_c, \theta_c) - B(t_c, a_n, 2, t_c, \theta_c) \quad (38)$$

The equations for the case where $t_c < t_o$ are obtained from Eqs. (36) through (38) by setting $t_c = t_o$. Similarly, for $t_c > t_f$, set $t_c = t_f$.

The point of closed approach is obtained by minimizing the performance index

$$d^2 = x_f^2 + y_f^2. \quad (39)$$

This is accomplished by solving the algebraic equation

$$\frac{\partial d^2}{\partial t_f} = 0 \quad (40)$$

by bisection. The derivative in (40) can be taken analytically. To verify that the solution is a minimum, the second derivative $\partial^2 d^2 / \partial t_f^2$ is checked.

Numerical Results

The algorithm followed in computing a trajectory is the following:

- Given V_T , ϕ , V_{Mo} , $a_{t,\max}$, $a_{t,\min}$, t_o , x_o , y_o , and t_c , compute θ_{ZA} . This determines whether $a_n > 0$ or $a_n < 0$.

2. Given $a_{n,\max}$ and θ_o , determine whether or not an intercept can occur.

3. If an intercept is possible, calculate the switch time for $x_f = y_f = 0$.

4. If an intercept is not possible, calculate the final time for closest point of approach.

Some results have been obtained for the following conditions:

$$V_T = 500 \text{ ft/sec}, \quad 0 \leq \phi \leq 180 \text{ deg}, \quad V_{Mo} = 1,000 \text{ ft/sec} \\ a_{t,\max} = 25 \text{ g's}, \quad a_{t,\min} = -10 \text{ g's}, \quad t_o = 0 \text{ sec}, \\ x_o = 5,000 \text{ ft}, \quad y_o = 0 \text{ ft}, \quad t_c = 2.6 \text{ sec}, \\ a_{n,\max} = 100 \text{ g's}, \quad -70 \leq \theta_o \leq 70 \text{ deg} \quad (41)$$

which are typical of a conceptual bank-to-turn missile engagement. The limit on θ_o is representative of the field of view of a passive seeker. Trajectories are presented in Figs. 3 through 7 for different values of ϕ primarily to illustrate that the computation process is valid.

The time-to-go algorithm has been tested in a six-degree-of-freedom simulation of a bank-to-turn missile. The missile velocity vector is projected onto the plane of the line of sight vector and the target velocity vector; the time-to-go is calculated in this plane; and the result used as the time-to-go for three-dimensional flight. Miss distance results are shown in Table 1 for a number of engagement scenarios. The results labeled Case I are for the case where time-to-go is calculated as range divided by closing speed. The results labeled Case II are for the case where bang-bang trajectories are used to calculate time-to-go. In six engagements, the bang-bang formula allows the missile to hit the target (within 10 ft) when the range-over-closing-speed formula allowed a miss. Only once does the reverse happen. In all other engagements both formulas generate a hit or a miss simultaneously. Overall, the bang-bang formula (Case II) provides better results.

Discussion and Conclusions

A method for predicting time-to-go for homing missiles using a linear-quadratic guidance rule has been developed. It is based on minimum-time trajectories for scenarios when intercepts are possible and on minimum-miss-distance trajectories when intercept trajectories are not possible in a reasonable time. The missile velocity is variable in that the tangential acceleration has a known positive value when thrusting and a known negative value when coasting and the engine cutoff time is known. On the other hand, the target velocity is assumed constant.

In general, the optimal trajectories are composed of constant normal acceleration segments. For an intercept trajectory the control is maximum normal acceleration followed by zero normal acceleration. For a minimum-miss trajectory, the control is maximum normal acceleration all the way. Numerical results for the time-to-go algorithm have been presented to verify the computational procedure. Also,

the time-to-go algorithm has been tested in a six-degree-of-freedom simulation. In general, the proposed time-to-go algorithm produces better miss distances than those obtained from the range-over-closing-speed method.

Acknowledgement

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References

- Hull, D. G., Speyer, J. L., and Burris, D. B., "A Linear-Quadratic Guidance Law for Dual Control of Homing Missiles," Paper No. 87-2381, AIAA Guidance, Navigation, and Control Conference, Monterey, California, August, 1987, pp. 551-559.
- Riggs, T. L., Jr., and Vergez, P. L., "Advanced Air-to-Air Missile Guidance Using Optimal Control and Estimation," AFATL-TR-81-56, Air Force Armament Laboratory, June, 1981.

Table 1: Simulation Results

Off-Boresight Angle (deg)	Launch Range (ft)	Aspect-Angle (deg)	Case I Miss Distance (ft)	Case II Miss Distance (ft)
0	7000	0	0.156	0.219
0	7000	30	0.695	0.266
0	7000	60	1.205	0.532
0	7000	90	1.318	0.823
0	7000	120	2.560	0.342
0	7000	150	0.734	3.199
0	7000	180	0.352	1.521
0	3000	0	0.995	0.218
0	3000	30	1.685	0.306
0	3000	60	1.401	1.497
0	3000	90	2.524	2.563
0	3000	120	13.52	5.354
0	3000	150	4.219	11.82
0	3000	180	0.585	4.562
40	7000	0	359.2	1159.
40	7000	30	259.7	1112.
40	7000	60	0.164	0.636
40	7000	90	0.922	0.532
40	7000	120	1.944	0.758
40	7000	150	1.016	0.672
40	7000	180	0.358	1.609
40	3000	0	2104.	4.084
40	3000	30	2895.	0.640
40	3000	60	2809.	0.198
40	3000	90	2519.	0.347
40	3000	120	1867.	0.490
40	3000	150	687.3	787.5
40	3000	180	224.5	396.0

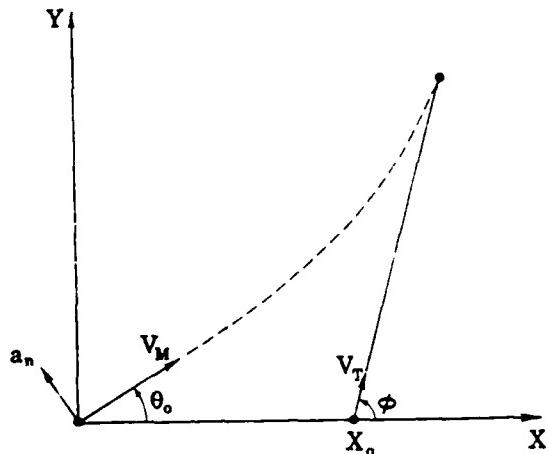


Fig. 1 Intercept geometry

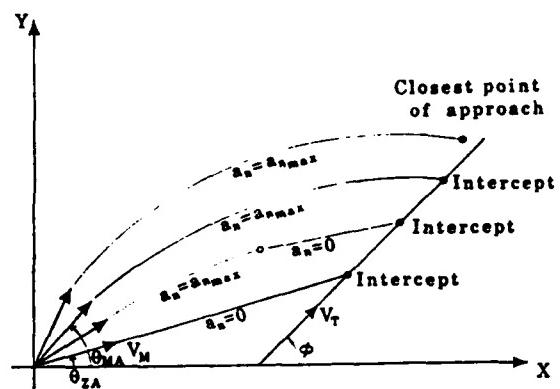


Fig. 2 Typical Trajectories

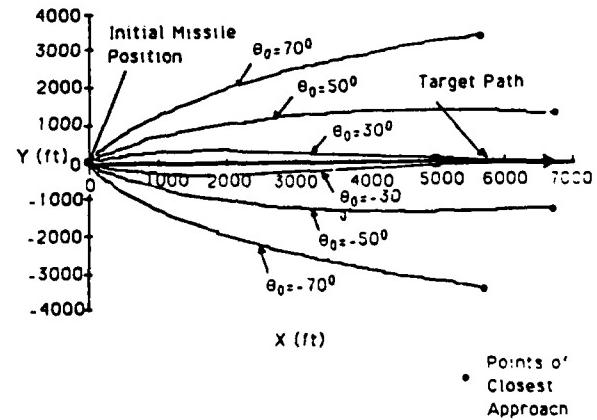


Fig. 3 Trajectories for $\phi = 0$ deg

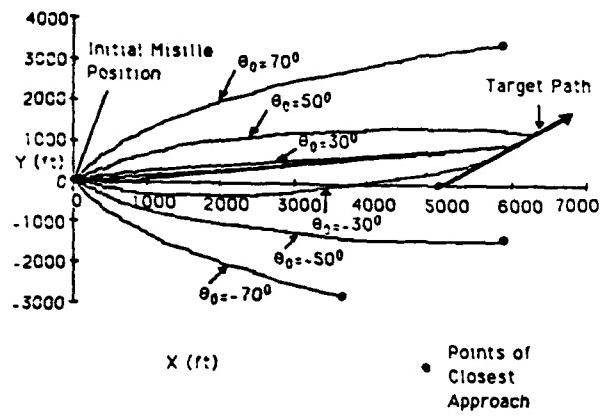


Fig. 4 Trajectories for $\phi = 45$ deg

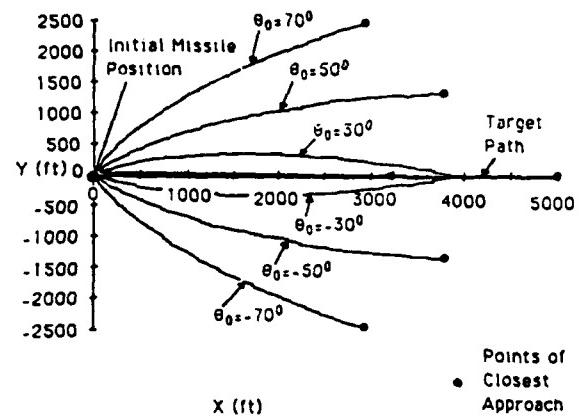


Fig. 7 Trajectories for $\phi = 180$ deg

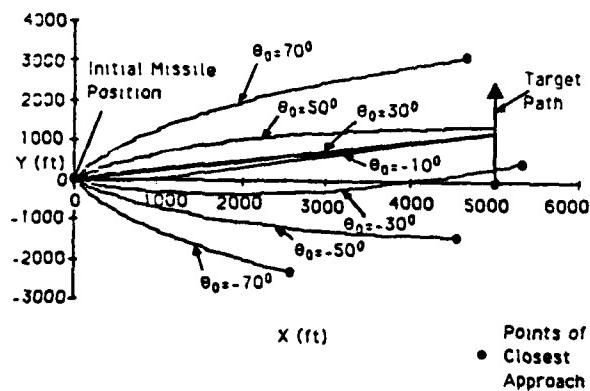


Fig. 5 Trajectories for $\phi = 90$ deg

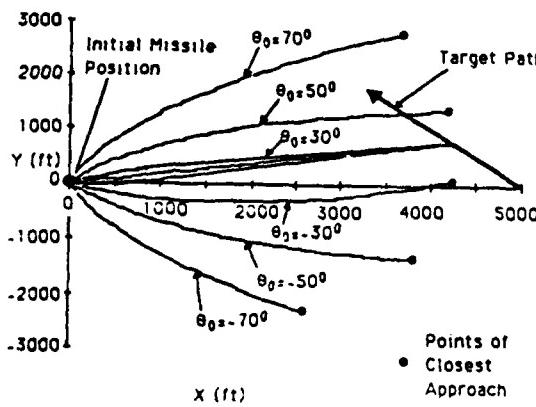


Fig. 6 Trajectories for $\phi = 135$ deg

Classification	Confidential
Declassification	1983
Supplementary Classification	Unclassified
By	
Classification/	
Amendability Codes	
Initial and/or	
Dist. Physical	

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